功能梯度材料之破裂力學回顧

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Briefly Review of Fracture Mechanics Applied in Functionally Graded Materials

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1. 前言

西元 1984 年，為了提高火箭發動機結構金屬本體與陶瓷鍍層間的結合強度並同時降低熱應力，首次製造出組織呈連續變化形式的耐熱材料[1]；日本科學技術廳航空宇宙研究所於 1987 年提出了梯度材料的概念，其基本構想是在原有陶瓷披覆層與金屬基底材料間介入一個過渡層材料，該材料的引入使得原有兩種巨觀性質完全不同的材料得以用一個連續變化的材料性質連接，藉由人為方式控制此過渡層材料的成分與微觀結構，可以有效避開原有母材間差異甚大的各種性質，不但降低因性質驟變所產生的界面應力過大問題，也充分降低了過高的溫度梯度所造成的熱應力集中。故開始有人提出「功能梯度材料」(Functionally gradient material)的名稱且相關研究陸續出現。西元 1990 年於日本仙台舉辦首屆國際功能梯度材料學術會議，此後每兩年舉行一次。為了更充分反映功能梯度材料的概念，1994 年第三屆會議時決定將功能梯度材料正式定名為 Functionally Graded Material (簡稱 FGM)，並自第四屆會議起使用[1]。就傳統材料的機械性質而言，金屬材料通常具有高強度、高韌性、高熱傳導係數等特點。但大多數金屬常在高溫、酸、鹼等腐蝕性強的作業環

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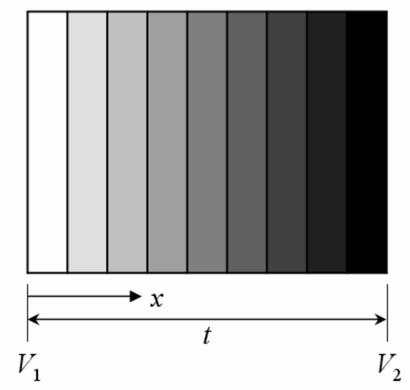
境下發生不同的失效問題，例如疲勞、裂斷、應力腐蝕、潛變、永久變形等。而非金屬材料中常被使用的陶瓷類材料則具有耐高溫、抗氧化、抗腐蝕等優點。但同樣地，陶瓷材料也面對著諸如質脆，硬度過高不易加工，無延展性等缺點。因此單一材料的性質勢必已經無法滿足現代工業多方面之功能需求。舉例而言，航太工業中的太空梭與火箭發射器本體結構雖由金屬製成，但為了避免在通過大氣層時不致因為過大的摩擦產生的高熱而遭到熔毀，需要在本體上塗佈上一層特製陶瓷塗層。其他如核能反應爐及燃氣渦輪機等可能處於高熱作業環境下的大型機械，也都要借助陶瓷的耐高溫與抗氧化特點，才能充分發揮其工作性能。為了達成這個目的，最常使用的方式即是以金屬材料為基材，在其外側鍍上一層陶瓷微顆粒材料，形成一個熱屏障表層(Thermal barrier coating)[2]。然而，此種複合材料是由二個不同種材料組成，在界面上因為兩者的材料性質諸如楊氏係數、降伏及抗拉強度、熱膨脹係數、熱傳導係數、延性與韌性等具有高度的差異性或不連續性，常在承受外加負荷後形成相當高的界面應力，若兩者之間的結合強度不足，常發生諸如裂縫、甚至脫層等界面失效現象，成為整組複合材料結構中最為脆弱的一部份。因此若能有效將兩結合材料間的材料性質差異降至最低，或者設法讓其突然的差異改為連續性變化，將可以減緩界面應力，進而降低破壞失效等發生的機率。

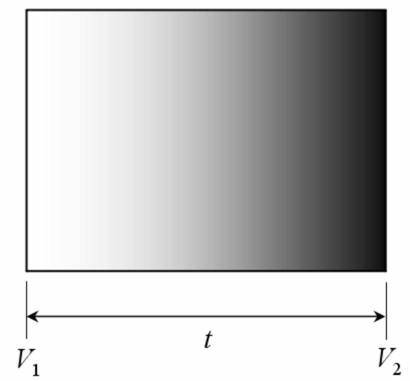
2. 何謂功能梯度材料

自然界中所有的動植物為了適應生存環境而經過漫長時間演化，逐漸形成各種不同天然的功能梯度材料，包括植物類的竹子，樹木，動物類的甲殼及骨骼等。以微觀結構而言，功能梯度材料是由二種或多種材料以不同的體積比例混合而成的複合材料，其材料特性在宏觀上卻呈現出

逐漸變化(梯度）性質。工程師可以用不同的材料組成與結構，以靈活的巧思，設計優化的元件，達到預期的功能需求。廣義的功能梯度材料，包含多層梯度材料及連續梯度材料，以一維結構而言可以分別以圖 1(a)及圖 1(b)說明。事實上，功能梯度材料可以擴充至二維及三維結構。本文僅針對連續梯度材料做簡單介紹。



(a)



(b)

圖 1　一維功能梯度材料示意圖 (a)片段(piecewise)函數結合；(b)連續函數結合

3. 功能梯度材料的製程

由於本文著重於如何以破裂力學評估含裂紋功能梯度材料的安全性，因此對於如何製作功能梯度材料，不做詳細描述，讀者可以參考過去的國際研討會記錄或期刊論文。詳細資料請參考文獻[3,4]。

4. 功能梯度材料的應用範圍

梯度材料的應用領域甚為廣泛，包括航空、太空、原子反應爐、燃氣渦輪機、鍋爐等承受劇烈反覆熱負載的場合，國際間已經公認功能梯度材料的研發屬於國防科技的核心關鍵技術。而且，在電信、光纖通訊、生醫工程、微機電系統甚至新興的奈米材料工程都可有其應用的前景。涵蓋研究的領域從機械、電機、電子、材料科學、化工、能源轉換、光學、核能、醫藥到醫學工程等。

5. 功能梯度材料破裂力學的研究發展

不同於一般之均勻材料，功能梯度材料破裂力學行為不僅受外加負載、結構元件幾何形狀與裂縫存在的位置及大小的影響，更與材料的非均勻性參數有著密切關係。因此，功能梯度材料破裂力學的力學解析變得較為複雜，以數學而言，僅有少數的非均勻分佈函數可以得到閉合解。

由於功能梯度材料的材料係數不再是常數，雖然依舊適用均質性材料的各統御方程式，然而卻將產生一些對於材料係數的偏微分項，增加解題困難度，也因此大大限制了材料性質的變化形式，經眾多學者研究發現，其中包括幾種特定的函數形式始可用數學解析方式求解[5]，包括

(1) 將材料性質假設為空間座標的冪次方函數(power function)，即

$$f(x_i) = f_0 x_i^{k_1} \quad (0 \leqq k_1 < 1) \tag{1}$$

其中$f(x_i)$代表空間座標中任意位置處的材料性質，x_i代表任意座標系統，如直角座標系、極座標系或圓柱座標系等；f_0則代表空間座標系座標原點位置處的材料性質。相關文獻如改變楊氏係數[6]、改變剪力係數[7-10]、改變密度[10]等研究，其缺點在於無法描述座標原點處的材料性質。

(2) 將材料性質假設為空間座標的多項式函數(polynomial function)，即

$$f(x_i) = f_0 (1 + k_2 |x_i|)^{k_3} \tag{2}$$

此型函數係由式(1)衍生而來，其中 k_2 與 k_3 為任意數，當 $k_3 = 1$ 時視為線性變化，當 $k_3 = 2$ 時則為拋物線變化。此處多項式前方加上一個常數 1 的目的在於避免發生如式(1)般材料性質於空間原點處消失（等於零）的現象。相關文獻如改變楊氏係數[11]、改變剪力係數[5,12-19]、改變普松比[14]等研究。

(3) 將材料性質假設為空間座標的指數型函數，即

$$f(x_i) = f_0 \exp(\alpha_i |x_i|) \tag{3}$$

其中 α_i 為控制梯度變化方向的非均質參數(nonhomogeneous parameter)。此類型的材料性質形式由於不會發生如上述般材料性質於空間原點處消失的現象，且指數函數微分處理較其他型式函數簡易，因此廣為最多人採用。相關文獻如改變楊氏係數[19, 20, 21-43]、改變剪力係數[18, 44-60]、改變普松比[20, 24, 25, 27, 30, 37]、改變熱膨脹係數與熱傳導係數[20, 27, 33, 35, 37, 53]及改變密度[18, 21, 39, 41, 43, 49, 60]等。

(4) 將材料性質假設為空間座標的雙曲線型函數(hyperbolic function)，即

$$f(x_i) = f_0 \tanh^2(\alpha_i x_i + \beta_i) \tag{4}$$

其中 α_i 為控制梯度變化方向的非均質參數，β_i 則為確保材料性質不致於消失。此型主要由指數型函數變化而來，具有與指數型函數相同的優點，相關文獻有[47]。

除了上述這幾大類外，亦有研究係先將材料性質假設為某一空間座標的未定函數(undetermined function)，經由數學解析處理方式後，根據所得的微分方程式的項再予以代入幾個相符的函數來做後續運算。屬於此類型的文獻有[63-66]。

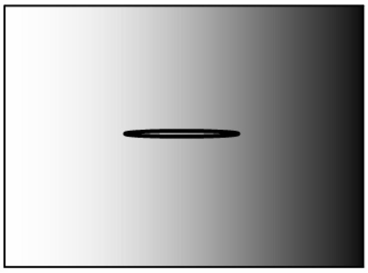
另外有一類研究並不預先假設材料性質的函數型式，而係假設整個梯度材料結構為許多薄層材料所組成，每一層具有相異的材料性質，以此來模擬材料性質逐漸變化的特性。相關文獻包括[67-75]。

在早期的研究中，大多同時將材料勁度係數(stiffness coefficient)及普松比假設為隨著相同的空間座標變化函數。文獻[33]研究無限平面中的裂縫問題，認為普松比的變化對於應力強度因子的影響可忽略不計，文獻[63]將楊氏係數與普松比同時假設為極座標系中兩個座標的任意函數，經由特徵函數展開式(eigenfunction expansion)求得裂縫端的應力奇異場並以有限元素法求得其應力強度因子，亦得知若將普松比假設為常數則不會影響裂縫端附近的奇異場行為。故後續諸多研究為了數學運算便利起見，多根據此結論將普松比設為常數。

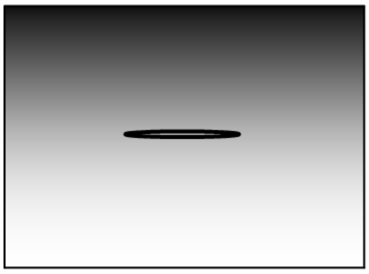
本文針對功能梯度彈性材料和功能梯度壓電材料，在破裂力學方面最近幾年的研究發展，包括靜態分析、動態分析、熱變形分析。由於引用的文獻相當多，僅將各類型的研究系統歸類，並不針對各篇論文詳細解說。

5-1 功能梯度彈性材料(FGM)

過去功能梯度彈性材料的裂縫問題研究相當普遍，依照裂縫與材料性質變化方式的關係，如圖 2 所示，可以分為材料性質梯度變化方向與裂縫線(crack line)平行[14, 16, 21, 23, 26, 28, 32-35, 37, 39, 42, 43, 45, 46, 51, 53, 55, 58, 72, 74-80]。材料性質梯度變化方向與裂縫線垂直[5, 10, 20, 24, 25, 27, 29, 30, 36, 40, 41, 47, 49, 52, 54, 56, 57, 59, 60, 68-71, 73, 81, 82]。材料性質同時為兩個座標軸的函數，既不與裂縫線平行亦不與其垂直，包括[44, 48, 50]，另外也有文獻研究討論正交性(orthotropic)功能梯度材料內的裂縫問題，如[29, 36, 57] ，沿著兩座標軸的材料性質以不同的非均質參數變化。



(a)

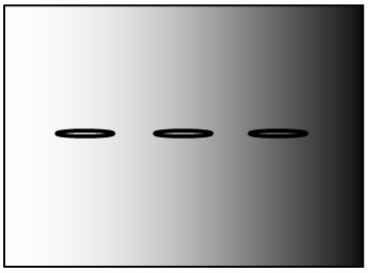


(b)

(c)

圖 2　功能梯度彈性材料結構內含裂縫示意圖 (a)材料性質梯度變化方向與裂縫線平行；(b)材料性質梯度變化方向與裂縫線垂直；(c)材料性質梯度變化為任意方向

依照裂縫的數目而分，可以分為單一裂縫或多個裂縫同時存在，後者又分為共線裂縫(collinear cracks)[32, 33, 46, 73, 78]與平行裂縫(parallel cracks)[51, 68, 70, 71, 76, 83]，如圖 3 所示。若依照裂縫形狀而分，則可分為直線型裂縫(straight crack 或 Griffth crack)、圓幣型裂縫(penny-shaped crack)[15, 16, 71]及圓柱型裂縫(cylindrical crack)[10]。



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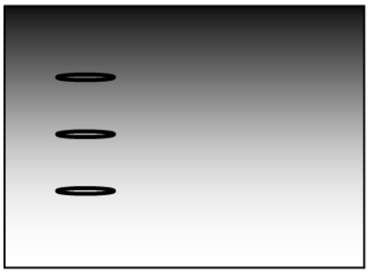

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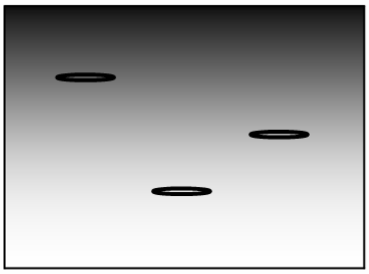

(c)

圖 3　功能梯度材料結構內含多個裂縫形式示意圖 (a)共線裂縫；(b)規則排列型平行裂縫；(c)不規則排列型平行裂縫

對於材料結構的外型而言，可以分為單一功能彈性梯度材料全平面問題[13, 15, 18, 23, 29, 36, 41, 48, 50, 57, 58]，單一功能彈性梯度材料半平面問題[59, 83]，兩個功能梯度彈性材料半平面結合問題[44, 45, 64]，功能梯度彈性材料與均質材料半平面結合問題[24, 26]，單一功能梯度彈性材料條板(strip)問題[14, 15, 20, 27, 38-40, 43, 53, 62, 64, 65, 67, 69]，多個條板與條板結合問題[28, 56, 78]、條板與平面結合問題[25, 32, 33, 42, 46, 47, 49, 60]以及鍍層(coating)與基層(substrate)問題[51, 52, 54, 76, 81, 82]。

針對材料結構所承受的外力，可區分為面內(inplane)問題與面外

(antiplane)問題。前者可以解出第一型(Mode I) [13, 14, 18, 20, 21, 23-29, 32, 34-37, 39, 41, 42, 48, 50, 52, 53, 55, 56, 59, 69-71, 73, 74, 78, 83]與第二型(Mode II) [18, 20, 24, 25, 27, 36, 41, 48, 50, 52, 56, 59, 70, 71, 73, 75]的應力強度因子；後者則解出第三型(Mode III)[5, 16-18, 43-47, 49, 51, 54, 57, 58, 60, 64, 68, 69, 75]的應力強度因子。

以上研究在破壞力學領域內有個重要的結論，當裂縫尖端的材料性質為連續變化，則裂縫端點的應力奇異性階數(stress singularity order)仍然維持-1/2，其對應的角函數(angular function)與均質材料相同[63, 66]，但應力強度因子則與材料非均質性有關。

此外，非均勻熱彈問題的研究中，熱負載(thermal loading)形式有溫度梯度[35, 37, 53, 62, 66, 67] 或熱傳量 [29, 71, 73, 82 44, 86, 88, 97]。材料熱膨脹係數或熱傳導係數等參數設為空間座標的函數，求出熱應力強度因子(thermal stress intensity factor)。

功能梯度彈性材料動態問題的研究有[5, 10, 17, 39, 43, 57, 60]，這些研究除了既有的勁度係數外，部分將材料密度設為空間座標的函數，可以求出裂縫端的動態應力強度因子。

5-2 功能梯度壓電材料(Functionally Graded Piezoelectric Material, FGPM)

壓電材料是一種廣泛使用於機電耦合領域之智慧型材料(smart material)，當其受到外加機械力作用時所產生之應變，會使材料產生電場(electric field)。反之，受到外加電場時，因內部的極化作用而產生相對應的位移[84-87]。然而，由於純壓電陶瓷(piezoelectric ceramic)相當脆，必需與其他金屬或複合材料結合，故其界面間也面臨許多問題，因此，功能梯度壓電材料逐漸受到重視。壓電常數(piezoelectric constant)與介電常數(dielectric constant)之梯度變化與彈性材料勁度係數的變化一致，同時

為了數學運算的便利性，絕大多數研究均將各材料性質假設為指數函數型式。

關於壓電材料的破壞力學研究已經相當普遍成熟，包括單一裂縫[88, 89]及多個裂縫[90-92]等。由於 FGPM 的逐漸發展，相關裂縫問題也開始引起許多學者注意，文獻[93]研究在外加熱負載作用下 FGPM 致動器中的裂縫問題，並使用能量密度因子理論預測裂縫成長方向。文獻[94]討論一個 FGPM 條板(strip)中的單一有限長度裂縫問題，其材料性質的梯度方向與裂縫面垂直且沿著條板寬度連續變化，該條板承受面外剪應力場與面內電場作用，得到與功能梯度彈性材料一致的結論，即裂縫尖端的應力與電位移奇異場與均質性壓電材料相同，但其應力強度因子與電位移強度因子受到材料梯度特性的強烈影響。文獻[95, 96]分別研究 FGPM 全平面及有限寬度條板內含單一裂縫的面外問題，但不事先假設其確切函數變化，而是根據所推得的偏微分方程式，找出幾個可能的函數型式。由於假設在面外剪應力作用之下，裂縫上下表面將可能互相接觸，故將裂縫面視為對電可滲透性(electrically permeable)，因此電位在裂縫表面連續，外加電負載對於裂縫尖端應力與電位移場無影響，即應力強度因子不受到壓電常數的影響。文獻[97]研究一個含有裂縫的 FGPM 條板與兩個純彈性材料條板接合的面外問題，其材料梯度變化方向與裂縫面平行。文獻[98]研究兩個 FGPM 條板接合的界面裂縫問題。文獻[99, 100]更進一步研究功能梯度壓電/壓磁材料(piezoelectric/piezomagnetic)中的承受面外剪力負載的裂縫問題。

關於 FGPM 的裂縫動態問題亦廣為學者感興趣。文獻[101, 102]分別研究 FGPM 全平面及條板中裂縫承受面外剪力及面內電位移的移動問題。文獻[103]研究接合於均質性壓電材料的 FGPM 條板面外裂縫擴展問題，討論不同的裂縫面邊界條件情形。文獻[104, 105]研究單一 FGPM 條

板面外裂縫問題。其他研究則是外加動態機械與電負載於裂縫面上來解出裂縫端點的奇異場行為與應力及電位移強度因子，包括全平面[106, 107]、單一條板[108-110] 、條板與半平面結合[111] 、條板與均質壓電材料結合[112, 113] 、條板與純彈性材料結合[114]及含多個平行裂縫[115]等問題。

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